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Remarks on the non-compact spin systems

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Abstract. For an abstract thermodynamically well behaved local specification describing a lattice spin system with non-compact state space we give a short proof of the independence of the limiting thermodynamics on the typical boundary conditions. This general theorem is then applied to superstable and regular spin systems studied by Lebowitz and Presutti to simplify and clarify their proof. Another application gives the uniqueness theorem for the limiting Gibbs phase for a class (in general non-superstable) of lattice systems with unbounded spins.

1. Introduction

There is a general belief that the infinite-volume thermodynamics of given statistical mechanical systems at thermal equilibrium should not depend on the particular choice of the typical boundary data. This has been rigorously established in a large class of statistical mechanical systems [1, 2, 3]. However, there are some main simplifying assumptions of the existing proofs, namely the compactness of the configurational space and short range of the interactions.

In this paper we will solve the problem of non-compactness of the corresponding configurational space by a simple probabilistic argument. In § 2 of the present paper we will consider a general class of local specifications describing lattice and classical spin systems. Under certain mild and natural assumptions the independence of the limiting thermodynamics on the typical boundary conditions is proved.

Section 3 includes some applications of the general theorem proved in § 2. The first application is to greatly simplify and clarify the corresponding proof of Lebowitz and Presutti [4], which works for superstable and (strongly) regular interactions only. We find the original proof of Lebowitz and Presutti to be rather complicated and long. Our new proof of their result is much simpler and works under weaker assumptions than those given in [4] (see note added in proof in [4]).

As a second application we consider a class of models which corresponds to the trigonometric perturbation of the Gaussian lattice models. The result regarding independence of the limiting free energy density on the typical boundary conditions is used together with some correlation inequalities of the Ginibre type to show the uniqueness of the limiting, translationally invariant Gibbs state for every regular value of the coupling constant. For superstable Gaussian spins this has been proved previously by us in [5], but our new proof also works without any superstability assumption.

Similar problems have been considered by Bellisard and Hoegh-Kröhn in [6]. However, models from our second example only partially belong to the class of local specifications for which the methods of [6] can be applied.

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Other applications of the method developed here are included in [7] and some others are in preparation.

2. General formulation of the van Hove theorem

Let us consider a classical lattice system defined on the unit lattice $\mathbb{Z}^d$. At each site $r \in \mathbb{Z}^d$ there is an associated random variable s_r taking values in some Polish space (X, ρ), where we denote by ρ a metric in X. The configuration space of the system is thus

$$\Omega_\infty = \mathop{\times}_{r \in \mathbb{Z}^d} (X, \rho)_r \tag{2.1}$$

where $(X, \rho)_r$ are identical copies of (X, ρ) attached to every lattice point r. The space Ω_∞ is equipped with the product topology. The corresponding Borel σ-algebra in Ω_∞ will be denoted by β_∞. Similarly with every subset $\Lambda \in \mathbb{Z}^d$ we can associate the corresponding configuration space Ω_Λ and a Borel σ-algebra β_Λ respectively. Elements of Ω_∞ will be denoted by s and its canonical restriction to Ω_Λ as s_Λ.

Let us assume that there is given '*a priori*' Borel measure $\lambda(\mathrm{d}x)$ on the configuration space (X, ρ). The family of bounded subsets of a lattice $\mathbb{Z}^d$ is denoted by $b(\mathbb{Z}^d)$ and the collection of all sequences $(\Lambda_n)_{n=1,\ldots,}$ of bounded, connected and one-connected subsets $\Lambda_n \subset b(\mathbb{Z}^d)$, which tend to $\mathbb{Z}^d$ monotonously and by inclusion, will be denoted by $c(\mathbb{Z}^d)$. The subset $c^{\mathrm{VH}}(\mathbb{Z}^d) \subset c(\mathbb{Z}^d)$ consists of van Hove-type sequences.

Any collection $\Pi = (\Pi_\Lambda(\; , \;))_{\Lambda \in b(\mathbb{Z}^d)}$ of probabilistic kernels defined on $\beta_\infty \times \Omega_\infty$ will be called a local specification iff the following conditions are fulfilled.

There exists a Borel subset $\Xi \subset \Omega_\infty$:

(1s 1) $\forall \Lambda \in b(\mathbb{Z}^d) \forall t \in \Xi \;\; \Pi_\Lambda(\mathrm{d}s_\Lambda / t)$ is a probability measure on β_∞, such that its restriction to β_{Λ^c} coincides with the point measure δ_t.

(1s 2) $\forall \Lambda \in b(\mathbb{Z}^d) \forall A \in \beta_\infty \forall t \in \Xi \;\; \Pi_\Lambda(A/t)$ is β_{Λ^c} measurable.

(1s 3) $\forall \Lambda_1, \Lambda_2 \in b(\mathbb{Z}^d)$: $\Lambda_1 \subset \Lambda_2 \;\; \Pi_{\Lambda_2} \circ \Pi_{\Lambda_1} = \Pi_{\Lambda_2}$.

Any probabilistic, cylinder set, Borel measure μ on $(\Omega_\infty, \beta_\infty)$, will be called a Gibbs measure corresponding to a given local specification Π iff

(i) $\mu(\Xi) = 1$

(ii) $\forall \Lambda \in b(\mathbb{Z}^d) \;\; \mu \circ \Pi_\Lambda = \mu$.

The set of all Gibbs measures corresponding to a given specification Π will be denoted by $\mathscr{G}(\Pi)$. For a general discussion of the questions of non-emptiness of the set $\mathscr{G}(\Pi)$, the consistency of the above definitions and the general properties of the set $\mathscr{G}(\Pi)$ we refer to [8–13]. Now we state our assumptions about Π from which we will be able to deduce our main result stated as theorem 2.1 below.

Let $\mathbf{O}$ be some fixed point in X. By $\mathbf{O}$ we denote an element in Ω_∞ defined as: $(\mathbf{O})_r = \mathbf{O}$ for every $r \in \mathbb{Z}^d$. Let us denote by $\mathscr{G}^{\mathbf{O}}(\Pi)$ the set of all (weak) (sub-)limits (assuming $\mathbf{O} \in \Xi$)

$$\lim_{n \to \infty} \Pi_{\Lambda_n}(\cdot \,|\mathbf{O}) \qquad \text{that are supported on } \Xi$$

as (Λ_n) ranges over the set $c(\mathbb{Z}^d)$.

It follows from the definition of $\mathscr{G}(\Pi)$ and $\mathscr{G}^{\mathbf{O}}(\Pi)$ that $\mathscr{G}^{\mathbf{O}}(\Pi) \subseteq \mathscr{G}(\Pi)$.

Assumption 0 (thermodynamic stability). The set $\mathscr{G}^{\mathbf{O}}(\Pi)$ is non-empty. Moreover, for any $\Lambda \in b(\mathbb{Z}^d)$ the measure $\Pi_\Lambda(\cdot \,|\mathbf{O})$, when restricted to the σ-algebra β_Λ, is absolutely continuous with respect to the *a priori* measure $\lambda_\Lambda = \times_{r\in\Lambda}\lambda_{(r)}$ and the corresponding, unnormalised Radon–Nikodym derivative $g^{\mathbf{O}}_\Lambda(s_\Lambda)$ satisfies the bound:

$$e^{-O_1'(\Lambda)|\Lambda|} \leqslant \int_{X_\Lambda} \lambda_\Lambda(\mathrm{d}s_\Lambda) g^{\mathbf{O}}_\Lambda(s_\Lambda) \leqslant e^{O_1(\Lambda)|\Lambda|} \tag{2.1}$$

where O_1' and O_1 are some slowly varying functions of the volume $|\Lambda|$, i.e.

$$\lim_{n\to\infty} \frac{O_1(\Lambda_n)}{|\Lambda_n|} = \lim_{n\to\infty} \frac{O_1'(\Lambda_n)}{|\Lambda_n|} = 0 \tag{2.2}$$

for any $(\Lambda_n) \subset c^{\mathrm{VH}}(\mathbb{Z}^d)$.

Remark. Note that $g^{\mathbf{O}}_\Lambda$ is defined by

$$\Pi_\Lambda(s_\Lambda|\mathbf{O}) = \left(\int_{X_\Lambda} \lambda_\Lambda(\mathrm{d}s_\Lambda) g^{\mathbf{O}}_\Lambda(s_\Lambda)\right)^{-1} g^{\mathbf{O}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda). \tag{2.3}$$

Assumption 1 (existence of the free energy density with fixed external condition). For any $(\Lambda_n) \subset c^{\mathrm{VH}}(\mathbb{Z}^d)$, there exists a unique limit

$$\lim_{n\to\infty} \frac{1}{|\Lambda_n|} \int_{X_\Lambda} \lambda_\Lambda(\mathrm{d}s_\Lambda) g^{\mathbf{O}}_\Lambda(s_\Lambda) \equiv P^{\mathbf{O}}_\infty(\Pi) \tag{2.4}$$

which is finite and independent of the sequence $(\Lambda_n) \in c^{\mathrm{VH}}(\mathbb{Z}^d)$ chosen.

For a given $\Lambda \in b(\mathbb{Z}^d)$, let $(\Sigma_n) \subset c(\mathbb{Z}^n - \Lambda)$ be given and for any $s_{\Lambda^c} \in \Omega_{\Lambda^c}$ let us define

$$(s^{\mathbf{O}}_{\Sigma_n})_r = \begin{cases} s_r & \text{if } r \in \Sigma_n \\ \mathbf{O} & \text{if } r \notin \Sigma_n. \end{cases} \tag{2.5}$$

Let $(\psi_2, \psi_3, \ldots, \psi_N)$ be a sequence of monotonic decreasing functions defined on R^+ and such that

$$\sum_{\substack{A\subset\mathbb{Z}^d, |A|=k \\ A\ni 0(\in\mathbb{Z}^d)}} \psi_k(d^{\mathrm{t}}(A)) = A_k < \infty \qquad \forall 2 \leqslant k \leqslant N \tag{2.6}$$

where $d^{\mathrm{t}}(-)$ means the tree diameter of the given set $A \subset \mathbb{Z}^d$.

Assumption 2 (regularity of Π). For every $t \in \Xi$, the measure $\Pi_\Lambda(-|t)$ when restricted to β_Λ is then absolutely continuous with respect to the measure λ_Λ. Let us denote by $g^t_\Lambda(s_\Lambda)$ the corresponding (unnormalised Radon–Nikodym) derivative and define also

$$h^t_\Lambda(s_\Lambda) = \ln g^t_\Lambda(s_\Lambda). \tag{2.7}$$

Then for every $(\Sigma_n) \subset c(\mathbb{Z}^d - \Lambda)$, there exist an integer $N \geqslant 2$, a sequence of functions $(\psi_2, \psi_3, \ldots, \psi_N)$ with the properties as above, and a sequence of integers $P_2, \ldots, P_N$ such that for any $t \in \Xi$:

$$|h^{t^{\mathbf{O}}_{\Sigma_n}}_\Lambda(s) - h^{\mathbf{O}}_\Lambda(s_\Lambda)| \leqslant \sum_{k=2}^{N} \sum_{\substack{A=\{r_1,\ldots,r_k\}\subset\Lambda\cup\Sigma_n \\ A\cap\Lambda\neq\varnothing, A\cap\Sigma_n\neq\varnothing}} \psi_k(d^{\mathrm{t}}(A)) \left(\sum_{i=2}^{k} \rho(\mathbf{O}, s_{r_i})^{P_k}\right).$$

Moreover for any $(\Sigma_n) \subset c(R^d - \Lambda)$ and any $\mu \in \mathscr{G}^{\mathbf{O}}(\Pi)$

$$\lim_{n\to\infty} \mu(|\ln g^{t^{\mathbf{O}}_{\Sigma_n}}_\Lambda - \ln g^t_\Lambda|) = 0. \tag{2.9}$$

Assumption 3 (local behaviour of moments). For any $\mu \in \mathscr{G}^{\mathbf{O}}(\Pi)$ the set of moments

$$\{\mu(\rho(\mathbf{O}, s_r)^{P_2}), \ldots, \mu(\rho(\mathbf{O}, s_r)^{P_N}\}$$

exists and

$$\forall k = 2, \ldots, N: \sup_{\mu \in \mathscr{G}^{\mathbf{O}}(\Pi)} \sup_{r \in \mathbb{Z}^d} \mu(\boldsymbol{\rho}(\mathbf{O}, s_r)^{P_k}) = B_k < \infty. \tag{2.10}$$

For a given $\Lambda \in b(\mathbb{Z}^d)$ and $\boldsymbol{t} \in \Xi$ let us define the finite volume free energy density $P^{\boldsymbol{t}}_\Lambda(\Pi)$ by

$$P^{\boldsymbol{t}}_\Lambda(\Pi) = \frac{1}{|\Lambda|} \ln \int_{X_\Lambda} \lambda_\Lambda(\mathrm{d}s_\Lambda) g^{\boldsymbol{t}}_\Lambda(\mathrm{d}s_\Lambda). \tag{2.11}$$

Then we have the following.

Theorem 2.1. Let Π be a local specification for which assumptions 0–3 are valid. Then for any $\boldsymbol{t} \in \Xi$, any $(\Lambda_n) \in c^{\mathrm{VH}}(\mathbb{Z}^d)$ there exists a unique limit

$$\lim_{n\to\infty} P^{\boldsymbol{t}}_{\Lambda_n}(\Pi) \equiv P^{\boldsymbol{t}}_\infty(\Pi)$$

which is equal to $P^{\mathbf{O}}_\infty(\Pi)$.

Proof. On the space $(X_\Lambda, \beta_\Lambda)$ we define two probability measures $\mu^{\mathbf{O}}_\Lambda(\mathrm{d}s_\Lambda)$ and $\mu^{\boldsymbol{t}}_\Lambda(\mathrm{d}s_\Lambda)$ by the formulae

$$\mu^{\mathbf{O}}_\Lambda(\mathrm{d}s_\Lambda) = \frac{\exp h^{\mathbf{O}}_\Lambda(s_\Lambda)}{\int_{X_\Lambda} \lambda_\Lambda(\mathrm{d}s_\Lambda) \exp h^{\mathbf{O}}_\Lambda(s_\Lambda)} \lambda_\Lambda(\mathrm{d}s_\Lambda) \tag{2.12}$$

$$\lambda^{\boldsymbol{t}}_\Lambda(\mathrm{d}s_\Lambda) = \frac{\exp h^{\boldsymbol{t}}_\Lambda(s_\Lambda)}{\int_{X_\Lambda} \exp h^{\boldsymbol{t}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda)} \lambda_\Lambda(\mathrm{d}s_\Lambda) \tag{2.13}$$

for $\boldsymbol{t} \in \Xi$.

Then we have

$$\begin{aligned}\int_{X_\Lambda} \lambda_\Lambda(\mathrm{d}s_\Lambda) \exp[h^{\boldsymbol{t}}_\Lambda(s_\Lambda)] &= \frac{\int_{X_\Lambda} \exp h^{\boldsymbol{t}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda)}{\int_{X_\Lambda} \exp h^{\mathbf{O}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda)} \int_{X_\Lambda} \exp h^{\mathbf{O}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda) \\ &\geqslant \exp(\mu^{\mathbf{O}}_\Lambda(h^{\boldsymbol{t}}_\Lambda - h^{\mathbf{O}}_\Lambda)) \int_{X_\Lambda} \exp h^{\mathbf{O}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda)\end{aligned} \tag{2.14}$$

(by Jensen inequality with respect to the measure $\mu^{\mathbf{O}}_\Lambda(\mathrm{d}s_\Lambda)$).

Therefore we have

$$P^{\boldsymbol{t}}_\Lambda(\Pi) - P^{\mathbf{O}}_\Lambda(\Pi) \geqslant \frac{1}{|\Lambda|} \mu^{\mathbf{O}}_\Lambda(h^{\boldsymbol{t}}_\Lambda - h^{\mathbf{O}}_\Lambda). \tag{2.15}$$

Proceeding analogously we have

$$\begin{aligned}\int_{X_\Lambda} \lambda_\Lambda(\mathrm{d}s_\Lambda) \exp[h^{\mathbf{O}}_\Lambda(s_\Lambda)] &= \frac{\int_{X_\Lambda} \exp h^{\mathbf{O}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda)}{\int_{X_\Lambda} \exp h^{\boldsymbol{t}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda)} \int_{X_\Lambda} \exp h^{\boldsymbol{t}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda) \\ &\geqslant \exp(\mu^{\boldsymbol{t}}_\Lambda(h^{\mathbf{O}}_\Lambda - h^{\boldsymbol{t}}_\Lambda)) \int_{X_\Lambda} \exp h^{\boldsymbol{t}}_\Lambda(s_\Lambda)\lambda_\Lambda(\mathrm{d}s_\Lambda)\end{aligned} \tag{2.16}$$

from which it follows that

$$P_\Lambda^{\mathbf{O}}(\Pi) - P_\Lambda^{\prime}(\Pi) \geqslant \frac{1}{|\Lambda|} \mu_\Lambda^{\prime t}(h_\Lambda^{\mathbf{O}} - h_\Lambda^{t}). \tag{2.17}$$

Now, let us take $(\Lambda_n) \in c^{\mathrm{VH}}(\mathbb{Z}^d)$ and let a sequence $\Sigma_m(n) \in c(\mathbb{Z}^d - \Lambda_n)$ be given for a fixed n. From the regularity assumption 2 we obtain

$$\forall \mu \in \mathscr{G}^0(\Pi): \int_{\Omega_\infty} \mu(dt)|\mu_{\Lambda_n}^{\prime t}(h_\Lambda^{\mathbf{O}} - h^{\prime \mathbf{O}}_{\Sigma_m(n)})|$$
$$\leqslant \sum_{k=2}^{N} \sum_{\substack{A=\{x_1,\dots,x_k\} \subset \Lambda \cup \Sigma_m(n) \\ A \cap \Lambda \neq \varnothing, A \cap \Sigma_m(n) \neq \varnothing}} \psi_k(d^t(A)) \sum_{j=1}^{k} \int_{\Omega_\infty} \mu(\mathrm{d}t)(\mu_\Lambda^{\prime t}((\boldsymbol{\rho}(\mathbf{O}, s_{x_j})^{n_j})). \tag{2.18}$$

Applying the DLR equation (1s 3) and noting the definition of μ we obtain

$$\int_{\Omega_\infty} \mu_\infty(\mathrm{d}t)(\mu_{\Lambda_n}^{\prime t}((\boldsymbol{\rho}(\mathbf{O}, s_{x_j})^{P_j}) = \int_{\Omega_\infty} \mu_\infty(\mathrm{d}t)\rho(0, t_{x_i})^{P_j}. \tag{2.19}$$

Hence, by (2.18) we obtain

$$\lim_{n\to\infty} \frac{1}{|\Lambda_n|} \int_{\Omega_\infty} \mu(\mathrm{d}t)|\mu_{\Lambda_n}^{\prime t}(h_{\Lambda_n}^{\mathbf{O}} - h^{\prime \mathbf{O}}_{\Sigma_m(n)})|$$
$$\leqslant \lim_{n\to\infty} \frac{1}{|\Lambda_n|} \sum_{k=2}^{N} \sum_{\substack{A \subset \Lambda \cup \Sigma_m(n) \\ |A|=k, A\cap\Lambda \neq \varnothing \\ A \cap \Sigma_m(n) \neq \varnothing}} \psi_k(d^t(A)) \left(\sum_{i=2}^{N} B_k \right) = 0.$$

We conclude that there is a sub-sequence $(n') \subset n$ such that for μ-almost everywhere $t \in \Omega_\infty$ we have

$$\overline{\lim_{n'\to\infty}} \frac{1}{|\Lambda_{n'}|} \mu_{\Lambda_{n'}}^{\prime t}(|h_{\Lambda_{n'}}^{\mathbf{O}} - h^{\prime \mathbf{O}\,\Sigma_m(n')}_{\Lambda_{n'}}|) = 0. \tag{2.20}$$

For a given $\varepsilon > 0$ and n, we can take $m = m(\varepsilon, n)$ such that

$$\mu(|h^{\prime \mathbf{O}\,\Sigma_{m(\varepsilon,n')}(n')}_{\Lambda_{n'}} - h_{\Lambda_{n'}}^{t}|) < \varepsilon. \tag{2.21}$$

From this we obtain

$$\overline{\lim_{n'\to\infty}} \frac{1}{|\Lambda_{n'}|} \mu|(\mu_{\Lambda_{n'}}^{\prime t}(h_{\Lambda_{n'}}^{\mathbf{O}} - h^t\Lambda_{n'})|$$
$$\leqslant \overline{\lim_{n'\to\infty}} \frac{1}{|\Lambda_{n'}|} \mu(\mu_{\Lambda_{n'}}^{\prime t}(|h_{\Lambda_{n'}}^{\mathbf{O}} - h^{\prime \mathbf{O}}_{\Sigma_{m(\varepsilon,n')}(n')}|))$$
$$+ \overline{\lim_{n'\to\infty}} \frac{1}{|\Lambda_{n'}|} \int_{\Omega_\infty} \mu(\mathrm{d}t)|h_{\Lambda_h}^{t} - h^{\prime \mathbf{O}}_{\Sigma_{m(\varepsilon,n')}(n')}| \leqslant \varepsilon$$

where in the last step we have used the DLR equations (1s 3) again.

In a similar way we conclude that for any $(\Lambda_n) \in c^{\mathrm{VH}}(\mathbb{Z}^d)$ there exists a sub-sequence $(n'') \subset (n)$ such that for every $t \in \Xi$ we have

$$\overline{\lim_{n''\to\infty}} \frac{1}{|\Lambda_{n''}|} \mu_{\Lambda_{n''}}^{\mathbf{O}}(|h_{\Lambda_{n''}}^{t} - h_{\Lambda_{n''}}^{\mathbf{O}}|) = 0. \tag{2.23}$$

Taking now $(\Lambda_n) \subset c^{\mathrm{HV}}(\mathbb{Z}^d)$, using assumption 1 and (2.15) and (2.17) we finish the proof. QED.

3. Some applications

3.1. Simplification and clarification of the Lebowitz-Presutti proof

In this section we will consider a class of Gibbsian local specifications which corresponds to the superstable and (everywhere strongly) regular interactions. In the basic paper [4], Lebowitz and Presutti, using the Ruelle probability estimates [14], have proved several important facts about this class of systems. However, as was pointed out by Preston, only the assumptions of superstability and regularity are not sufficient to control the set of all tempered solutions of the corresponding DLR equations. The additional restrictions on the interactions to remedy this situation were proposed by Lebowitz and Presutti in [15].

For our purposes it is enough to assume only superstability and regularity of interactions. Then we can control elements of the set $\mathscr{G}^{\mathrm{O}}$ using probability estimates on the densities corresponding to the empty boundary conditions and such information is sufficient for all assumptions of theorem 2.1 to be valid. To be more concrete we will consider a spin system on $\mathbb{Z}^d$ with values in R^1, i.e. $\Omega_\infty = R^{\mathbb{Z}^d}$. The *a priori* Borel measure λ on R^1 is such that

$$\underset{\alpha>0}{\exists}: \quad \int e^{\alpha x^2}\lambda(\mathrm{d}x) < \infty \tag{3.1}$$

$$\underset{x_0\in R^1}{\forall}: \quad \lambda(\mathrm{d}x) \neq \delta(x-x_0). \tag{3.2}$$

The local specification $\Pi_\Lambda(s_\Lambda|t)$ is given by

$$\Pi(\mathrm{d}s_\Lambda|t) = (Z^t_\Lambda)^{-1}\exp[-U_\Lambda(s_\Lambda)]\exp[-U_\Lambda(s_\Lambda|t_{\Lambda^c})]\bigotimes_{i\in\Lambda}\mathrm{d}\lambda(s_i) \tag{3.3}$$

$$Z^t_\Lambda = \int_{\Omega_\Lambda}\lambda_\Lambda(\mathrm{d}s_\Lambda)\exp[-U_\Lambda(s_\Lambda)]\exp[-U_\Lambda(s_\Lambda|t_{\Lambda^c})] \tag{3.4}$$

where U_Λ fulfils the superstability condition with the constant $A<\alpha$ and the (2)-regularity condition with $k_1 = k_2 = 2$.

Then the probability estimates of Ruelle [14] are valid. From (3.2) and the probability estimates it follows that the set $\mathscr{G}^{\mathrm{O}}(\Pi_\Lambda)$ is non-empty and every $\mu\in\mathscr{G}^{\mathrm{O}}(\Pi_\Lambda)$ is supported on the set of tempered configurations $\Omega^{\mathrm{T}}_\infty$ which is defined by

$$\Omega^{\mathrm{T}}_\infty = \bigcup_{a=1}^{\infty}\Omega_\infty(a)$$

$$\Omega_\infty(a) = \{t\in\Omega_\infty | \underset{r_*}{\exists}\ \underset{|i|>r_*}{\forall}\ |t_i| < a\log|i|\}.$$

Repeating the argument of Ruelle [16], Lebowitz and Presutti proved that for any van Hove-type sequence $(\Lambda_n)\subset c(\mathbb{Z}^d)$, the unique thermodynamic limit

$$\lim_{n\to\infty}\frac{1}{|\Lambda_n|}\ln Z^{t=0}_\Lambda = P^\phi_\infty \tag{3.5}$$

exists and is independent of the particular choice of the van Hove sequence (see [4], lemma 2.5). From our theorem 2.1, we now have the following.

Theorem 3.1. Let the Gibbsian local specification $\Pi = (\Pi_\Lambda)$ fulfil (3.1-3.4) and obey superstability and the (2)-regularity condition. Then for any $(\Lambda_n)\in c^{\mathrm{VH}}(\mathbb{Z}^d)$, and $t\in\Omega^T_\infty$ the unique thermodynamic limit of P^t_Λ exists and $\lim_{n\to\infty}P^t_{\Lambda_n} = P^\phi_\infty$.

A similar result has been proved by Lebowitz and Presutti ([4], Theorem 3.1) using some rather complicated estimates (see appendix B of [4]). Lebowitz and Presutti had to assume some special geometry of regions (Λ_n) (see note added in proof in [4]) while our proof is simple and does not require any additional assumptions.

3.2. *Trigonometric perturbations of the Gaussian lattice fields*

Let ν_ρ be a stationary, Gaussian random field defined on the space $\Omega_\infty = R^{Z^d}$, with mean zero and spectral density $\rho(k)$, $k \in [-\pi, \pi]^d$ defined by

$$\begin{aligned}\nu_\rho(s_0 s_x) &= V(x)\\ &= (2\pi)^{-d} \int_{[-\pi,\pi]^d} \rho(k) e^{ikx} dk. \end{aligned} \tag{3.6}$$

The inverse matrix $(A_{x,y})_{x,y\in Z^d}$, to the matrix $(\hat{V})_{x,y} = V(x-y)$ is given by

$$a_{xy} \equiv a'_{x-y} \equiv (A)_{x,y} = (2\pi)^{-d} \int_{[-\pi,\pi]^d} \frac{e^{ik(x-y)}}{\rho(k)} dk. \tag{3.7}$$

In the following we will assume

$$\sum_{x\in Z^d} |a'_x| < \infty. \tag{3.8}$$

The explicit expressions for conditional expectation values $E_{\nu_\rho}\{-|\beta_{\Lambda^c}\}(t)$ are known and are given by the formulae

$$\begin{aligned}(\nu_\rho - \text{a.e. } t)\colon \Pi^0_\Lambda(-|t) &\equiv E_{\nu_\rho}\{-|\beta_{\Lambda^c}\}(t)\\ &= (Z^t_\Lambda)^{-1} \int_{\Omega_\Lambda} - \exp\Big(-\frac{1}{2}\sum_{i,j\in\Lambda} a_{i-j} s_i s_j\Big) \exp\Big(-\sum_{i\in\Lambda}\sum_{j\in\Lambda^c} a_{i-j} s_i t_j\Big) ds_\Lambda. \end{aligned} \tag{3.9}$$

The conditional partition function Z^t_Λ, for $\Lambda \in b(Z^d)$ can be computed by a simple Gaussian integration:

$$Z^t_\Lambda = (\det(A)_{x,y\in\Lambda})^{-1/2} (2\pi)^{|\Lambda|/2} \exp\Big(\frac{1}{2}\sum_{i,j\in\Lambda} V_\Lambda(i,j)\Big(\sum_{k\notin\Lambda} a_{j-k} t_k\Big)\Big(\sum_{k\notin\Lambda} a_{i-k} t_k\Big)\Big) \tag{3.10}$$

where V_Λ is the inverse of the $|\Lambda| \times |\Lambda|$ matrix $(a_{x,y})_{x,y\in\Lambda}$. The DLR equations corresponding to the local specification (Π^0_Λ) have been studied by Rosanov [17], Dobrushin [18] and Künsh [19] and the constructive description of the set $\mathscr{G}(\Pi^0)$ is quite well known. Every Gibbs state $\mu \in \mathscr{G}(\Pi^0)$ is a mixture of Gaussian fields with covariance V and a mean value $\mu(s_x) = m_x$ satisfying

$$\sum_{y\in Z^d} a'_y m_{x-y} = 0. \tag{3.11}$$

This equation has in general many non-constant solutions which lead to the existence of non-stationary Gaussian solutions of the DLR equation

$$\mu \circ \Pi^0 = \mu. \tag{3.12}$$

This is called in physics spontaneous breaking of translational invariance. If $\sum_{x\in Z^d} a'_x = \rho(0) < \infty$ then the only constant solution of (3.7) is $m_x = 0$. Rosanov [17] has shown that any stationary Gaussian field in $\mathscr{G}(\Pi^0)$ with mean zero has spectral measure of the form

$$(2\pi)^{-d}\left(\sum_{x\in\mathbb{Z}^d} a'_x \,\mathrm{e}^{\mathrm{i}kx}\right)^{-1} + \mathrm{d}F_s(k) \tag{3.13}$$

where $\mathrm{d}F_s(k)$ is an arbitrary measure concentrated on $\{k|\Sigma_{x\in\mathbb{Z}^d} a_x \,\mathrm{e}^{ikx}=0\}$.

In particular, it follows from this that the uniqueness of the stationary Gaussian field in $\mathscr{G}(\Pi^0)$, with finite second moment is guaranteed by the boundedness of $\rho(k)$ on $[-\pi, \pi]^d$.

Remark 1. For any $G(s)=G(s_\Lambda)\in L^2_{\nu_\rho}$ from the Martingale convergence theorem it follows that for any $(\Lambda_n)\subset c(\mathbb{Z}^d)$, $E_{\nu_\rho}(F|\beta_{\Lambda_n^c})$ converges in $L^2_{\nu_\rho}$ to $E_{\nu_\rho}\{F|\beta_\infty\}$, where $\beta_\infty=\bigcap_{n>0}\beta_{\Lambda_n^c}$. By theorem 1 of Rosanov it follows that β_∞ is trivial σ-algebra for ν_ρ. From this fact one can easily conclude (using the Martingale convergence theorem) that for any $(\Lambda_n)\in c(\mathbb{Z}^d)$, $V_{\Lambda_n}\to V$ uniformly on compacts in $\mathbb{Z}^d$.

Here we consider the perturbation of the Gaussian local specification Π^0 by the self-interaction

$$U_1(s_x)=z\cos\alpha s_x \pm m_0^2 s_x^2. \tag{3.14}$$

This means that we will consider a local specification (Π_Λ^{z,m_0}) of the following form:

$$\Pi_\Lambda^{z,m_0}(\mathrm{d}s_\Lambda|t)=(Z^t_\Lambda(z,m_0))^{-1}\exp\left(-\frac{1}{2}\sum_{i,j\in\Lambda} a_{i-j}s_is_j\right)\exp\left(-\sum_{i\in\Lambda}\sum_{j\notin\Lambda} a_{i-j}s_it_j\right)$$
$$\times\exp\left(\sum_{i\in\Lambda} z\cos\alpha s_i\right)\exp\left(\pm m_0^2\sum_{i\in\Lambda} s_i^2\right) \tag{3.15}$$

where $Z^t_\Lambda(z,m_0)$ is the corresponding conditional partition function. We denote by $\mathscr{G}(z,m_0)$ the set of solutions of the equation

$$\mu\circ\Pi_\Lambda^{z,m_0}=\mu. \tag{3.16}$$

Let $P_{m_0}(k)=a_0\pm m_0^2-\Sigma_{x\neq 0}\, a_k\mathrm{e}^{ikx}$. Then the local specification given by (3.9) is superstable iff $\inf_{k\in[-\pi,\pi]^d} P_{m_0}(k)\equiv A>0$ and A gives the best possible value for the superstability constant. From the results of [19] it follows then (for $z=0$) that every Gaussian solution of (3.7) is supported on the set of tempered spin configurations $\Omega^{\mathrm{T}}(R^d)$, even in the non-superstable case. For the Gaussian case (i.e. $z=0$) in order to have $\mathscr{G}(\Pi^{0,m_0})\neq\phi$ it is sufficient to have $P_{m_0}(k)\geqslant 0$ and $\int_{[-\pi,\pi]^d} P_{m_0}(k)^{-1}\,\mathrm{d}k<\infty$.

Lemma 3.2. For any $z\geqslant 0$ the set of solutions of (3.10) is non-empty, provided

$$\int_{[-\pi,\pi]^d} P^{-1}_{m_0}(k)\,\mathrm{d}k<\infty \qquad\text{and}\qquad P_{m_0}(k)\geqslant 0. \tag{3.17}$$

Proof. Let us denote by $s(\mathbb{Z}^d)$ the space of rapidly decreasing sequences on $\mathbb{Z}^d$ equipped with the nuclear topology induced by the sequence of norms $\|\ \|_m$ given by

$$\|s\|_m=\sup_{x\in\mathbb{Z}^d}|s_x|(1+|x|)^m. \tag{3.18}$$

Let $s'(\mathbb{Z}^d)$ be the strong dual of the space $s(\mathbb{Z}^d)$. Then the pair $(s(\mathbb{Z}^d), s'(\mathbb{Z}^d))$ forms a dual pair of nuclear spaces. From the assumption of the lemma and the Minlos theorem it follows that the functional

$$s(\mathbb{Z}^d)\ni\boldsymbol{\alpha}\to\Gamma(\boldsymbol{\alpha})\equiv\exp-\tfrac{1}{2}V(\boldsymbol{\alpha},\boldsymbol{\alpha}) \tag{3.19}$$

where

$$V(x)=\int_{[-\pi,\pi]} P_{m_0}(k)\,\mathrm{e}^{-\mathrm{i}kx}\,\mathrm{d}k \tag{3.20}$$

$$V(\boldsymbol{\alpha},\boldsymbol{\alpha})=\sum_{x,y\in\mathbb{Z}^d}\alpha_x V(x-y)\alpha_y \tag{3.21}$$

is then a Fourier transform of some probabilistic, Borel, cylindric set Gaussian measure μ_V^0 supported on the set $s'(\mathbb{Z}^d)$ and such that

$$\Gamma(\boldsymbol{\alpha})=\int_{s'(\mathbb{Z}^d)}\mu_V^0(\mathrm{d}s)\,\mathrm{e}^{\mathrm{i}(\boldsymbol{\alpha},s)}. \tag{3.22}$$

Using the Gaussian measure $\mu_V^0(\mathrm{d}s)$ we define its perturbation:

$$\mu_\Lambda(z|\mathrm{d}s_\Lambda)=(Z_\Lambda(z))^{-1}\exp\sum_{x\in\Lambda} z\cos\alpha s_x\mu_V^0(\mathrm{d}s) \tag{3.23}$$

$$Z_\Lambda(z)=\int_{s'(\mathbb{Z}^d)}\exp\sum_{x\in\Lambda} z\cos\alpha s_x\mu_V^0(\mathrm{d}s). \tag{3.24}$$

Then the following correlation inequalities are valid (see [20]):

$$\left.\begin{aligned}&\mu_\Lambda(z|\mathrm{e}^{t(s,\boldsymbol{\alpha})};\cos\alpha s_x)^{\mathrm{T}}\leqslant 0 &(3.25)\\ &\mu_\Lambda(z|(s,\boldsymbol{\alpha})^2;\cos\alpha s_x)^{\mathrm{T}}\leqslant 0 &(3.26)\end{aligned}\right\}\quad \forall t\in R^1$$

where $\mu_\Lambda(z|\cdot;\cdot)^T$ means the truncated expectation values. From these inequalities it follows that both $\mu_\Lambda(z|\mathrm{e}^{t(s,\boldsymbol{\alpha})})$ and $\mu_\Lambda(z|(s,\boldsymbol{a})^2)$ are monotonically decreasing in the volume Λ. Hence by the Vitali theorem the unique thermodynamic limits

$$\left.\begin{aligned}&\lim_{\Lambda\uparrow\mathbb{Z}^d}\mu_\Lambda(\mathrm{e}^{\zeta(s,\boldsymbol{\alpha})})\equiv\mu_\infty(z|\mathrm{e}^{\zeta(s,\boldsymbol{\alpha})}) &(3.27)\\ &\lim_{\Lambda\uparrow\mathbb{Z}^d}\mu_\Lambda((s,\boldsymbol{\alpha})^2)\equiv\mu_\infty(z|(s,\boldsymbol{\alpha})^2) &(3.28)\end{aligned}\right\}\quad\forall\zeta\in\mathbb{C}^1$$

exist and obey the bounds

$$\left.\begin{aligned}&|\mu_\infty(z|\mathrm{e}^{\zeta(s,\boldsymbol{\alpha})})|\leqslant\exp\left(\frac{(\mathrm{Re}\,\zeta)^2}{2}V(\boldsymbol{\alpha},\boldsymbol{\alpha})\right) &(3.29)\\ &|\mu_\infty(z|(s,\boldsymbol{\alpha})^2)|\leqslant V(\boldsymbol{\alpha},\boldsymbol{\alpha}). &(3.30)\end{aligned}\right\}\quad\forall\zeta\in\mathbb{C}^1$$

Applying remark 1 it is not difficult to conclude that the constructed measure $\mu_\infty(z|-)$ belongs to the set $\mathscr{G}(\Pi_\Lambda^{z,m_0})$. QED.

From this lemma it follows that the set $\mathscr{G}^0(\Pi^{z,m_0})$ (i.e. the set of Gibbs measures obtained with the empty boundary conditions) consists exactly of one element $\mu_\infty(z|\cdot)$.

By another simple correlation inequality of Fröhlich and Park [20]:

$$Z_\Lambda(z)\leqslant Z_{\Delta_1}(z)Z_{\Delta_2}(z) \tag{3.31}$$

valid for any $\Lambda\in b(\mathbb{Z}^b)$, $\Delta_1\cap\Delta_2=\varnothing$; $\Delta_1\cup\Delta_2=\Lambda$, we obtain the existence of the unique thermodynamic limit of the pressure corresponding to the empty boundary condition.

Lemma 3.3. For any $z \geqslant 0$, the unique thermodynamic limit

$$\lim_{n\to\infty} P^{\phi}_{\Lambda_n}(z) = P^{\phi}_{\infty}(z)$$

exists and does not depend on the sequence $(\Lambda_n) \in c(\mathbb{Z}^d)$ chosen.

An immediate application of theorem 2.1 gives the following result.

Lemma 3.4. Let $(\Lambda_n) \in c(\mathbb{Z}^d)$ and let us assume that (a_{ij}) fulfils the (2)-regularity condition with $k_1 = k_2 = 2$. Then for any $z \geqslant 0$ and $\mu_\infty(z|\cdot)$ almost everywhere $t \in \Omega_\infty$ the unique thermodynamic limit

$$\lim_{n\to\infty} P^{t}_{\Lambda_n}(z) = P^{t}_{\infty}(z)$$

exists and is equal to $P^{\phi}_{\infty}(z)$.

An interesting application of lemma 3.4 seems to be the following theorem.

Theorem 3.5. Assume that

(i) $\int_{[-\pi,\pi]^d} P^{-1}_{m_0}(k)\,\mathrm{d}k < \infty \qquad P_{m_0}(k) \geqslant 0.$

(ii) (a_{ij}) obeys the 2-regularity condition as above.

Let $z = z_0$ be a regular value for the infinite-volume free energy $P^{\phi}_{\infty}(z)$.

Then the set of Ξ-regular solutions which have translationally invariant first moment of the DLR equation (3.16) consists exactly of one element $\mu_\infty(z_0|-)$.

Proof. For the proof we use again μ^0_V integrations and some correlation inequalities of Pfister.

Let us start with the conditioned partition function. We use an abbrevation

$$\sum_{j\notin\Lambda} t_j a_{i-j} \equiv t_i(\Lambda^c) \tag{3.32}$$

$$\begin{aligned} Z^t_\Lambda(z, m_0) = \int_{\Omega_\Lambda} \mathrm{d}s_\Lambda \prod_{i\in\Lambda} \exp(-\tfrac{1}{2}m_0^2 s_i^2) \\ \times \exp(z\cos\alpha s_i)\exp\bigg(-\frac{1}{2}\sum_{i,j\in\Lambda} a_{ij}s_is_j\bigg)\exp\bigg(-\sum_{i\in\Lambda} s_i t_i(\Lambda^c)\bigg) \\ = (2\pi)^{|\Lambda|/2}\det((A^{m_0})_{i,j\in\Lambda})^{-1/2}\exp[\tfrac{1}{2}V^{m_0}_\Lambda(t_\Lambda(\Lambda^c), t_\Lambda(\Lambda^c))] \\ \times \int_{s'(\mathbb{Z}^d)} \mu^0_{\Lambda|\Sigma(\Lambda)}(\mathrm{d}s_\Lambda) \prod_{i\in\Lambda} \exp[z\cos\alpha(s_i + t_i(\Lambda^c))] \end{aligned} \tag{3.33}$$

where $V^{m_0}_\Lambda$ is the matrix inverse to the matrix

$$(A^{m_0})_{ij} = a_{ij} \pm \delta_{ij}m_0^2. \tag{3.34}$$

By a similar calculation we get the following expression for $\Pi^{z,m_0}_\Lambda(F)$:

$$\tilde{Z}^t_\Lambda(z, m_0)\Pi^{z,m_0}_\Lambda(F, t) = \int_{s'(\mathbb{Z}^d)} \tilde{\mu}^{m_0,t}_\Lambda(z|\mathrm{d}s_\Lambda)F(s_\Lambda, t_\Lambda(\Lambda^c)) \tag{3.35}$$

where

$$\tilde{\mu}_{\Lambda}^{m_0,t}(z|ds_{\Lambda}) = \tilde{Z}_{\Lambda}^{t}(z, m_0)^{-1} \prod_{i\in\Lambda} \exp[z\cos(\alpha s_i + t_i(\Lambda^c))]\mu_{|\Sigma(\Lambda)}(ds_{\Lambda}) \quad (3.36)$$

and

$$\tilde{Z}_{\Lambda}^{t}(z, m_0) = (2\pi)^{-|\Lambda|/2} \det(A^{m_0})_{ij\in\Lambda})^{-1/2} \exp[-\tfrac{1}{2}V_{\Lambda}^{m_0}(t_{\Lambda}(\Lambda^c), t_{\Lambda}(\Lambda^c))]. \quad (3.37)$$

Let us denote by $\tilde{\mu}_{\Lambda}^{O,t}(ds_{\Lambda}, ds'_{\Lambda})$ the tensor product of $\tilde{\mu}_{\Lambda}^{m_0,O}(z|ds_{\Lambda})$ and $\tilde{\mu}_{\Lambda}^{m_0,t}(z|ds'_{\Lambda})$. Then the following correlation inequality of Pfister [21] is valid:

$$\tilde{\mu}_{\Lambda}^{O,t}\left[\left(\prod_{i=1}^{n}\cos\alpha s_{x_i} - \prod_{i=1}^{n}\cos\alpha s'_{x_i}\right)\exp\delta\sum_{i\in\Lambda}\cos\alpha s_i\cos\alpha s'_i\right] \geq 0 \quad (3.38)$$

for any $n \geq 0$, $\delta \in R$.

Also the following correlation inequality of Fröhlich-Park [20] is valid: for any $n \geq 1$, integer and $\theta_i \in [0, 2\pi)$

$$\left|\mu_{\Lambda}^{t}\left(\prod_{i=1}^{n}\cos(\alpha s_i + \theta_i)\right)\right| \leq \mu_{\Lambda}\left(z\,\middle|\,\prod_{i=1}^{n}\cos\alpha s_i\right). \quad (3.39)$$

These two sequences of correlation inequalities lead to the following bootstrap principle. If

$$\lim_{\Lambda\uparrow Z^d} \mu_{\Lambda}^{m_0,t}(z|\cos\alpha s_i) = \mu_{\infty}(\cos\alpha s_0) \quad (3.40)$$

then for any $n, m \geq 1$ we have

$$\lim_{\Lambda\uparrow Z^d} \mu_{\Lambda}^{m_0,t}\left(z\,\middle|\,\prod_{i=1}^{n}\cos\alpha s_i\prod_{j=1}^{m}\cos\alpha s_j\right) = \mu_{\infty}\left(\prod_{i=1}^{n}\cos\alpha s_i\prod_{j=1}^{m}\cos\alpha s_j\right). \quad (3.41)$$

The assumed decay properties of a_{ij}((2)-regularity assumption), assumed translational invariance of $\lim_{\Lambda\uparrow Z^d}\mu_{\Lambda}^{m_0,t}(z|\cdot)$, regularity of z_0 and lemma 3.4 end the proof. QED.

It seems to be of some interest to describe the whole set $\mathscr{G}(\Pi_{\Lambda}^{z,m_0})$. The 'zero-temperature' ground states are labelled by solutions (in $s'(\mathbb{Z}^d)$) of the equation

$$m_0^2 s_i^2 + z\alpha\sin\alpha s_i + \sum_{j\in Z^d} a_{ij}s_j = 0. \quad (3.42)$$

The interesting question is whether there exist solutions of (3.42) which are 'non-trivially' periodic and whether such solutions lead to the stable ground states. If such a situation occurs then we hope to prove the existence of the crystalline order in such systems.

Assuming $a_{ij} \geq 0$ we have that FKG inequalities are valid. Then for the lattice scalar Bose fields [22] we can prove that the unique Gibbs measure $\mu_{\infty}(z|\cdot)$ described above is globally Markov. This suggests the possibility that the uniqueness theorem proved in [23] for the continuous, quantum, two-dimensional, Euclidean sine-Gordon fields can be presumably extended to the statement about the global Markov property for this class of quantum fields.

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